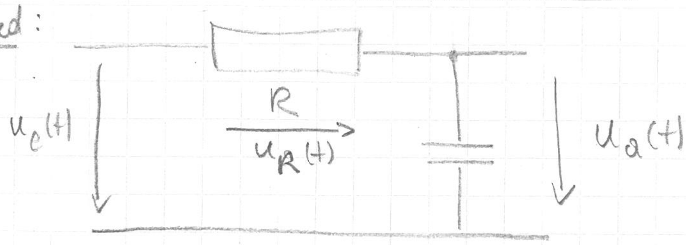
Modellierung von Input/Output-Systemen mittels gewöhnl. DGL

▷ RC-Glied:



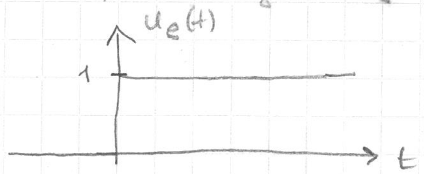
$$u_e = u_R + u_a$$

$$u_R = i R$$

$$i = C\,\dot{u}_a$$

$\Rightarrow$ $\underline{u_e = RC\dot{u}_a + u_a}$ ...lineare, inhomogene DGL 1. Ordg.

Bsp.: Sprungantwort



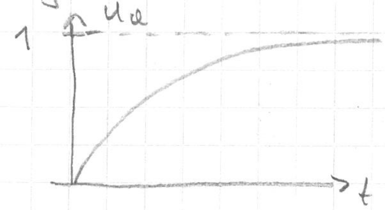
$u_a(0) = 0$

vgl. LV AM $\boxed{\begin{array}{l} \dot{y} + a y = b \quad \text{hat Lsg.} \\ y(t) = y(0)\, e^{-at} + \frac{b}{a}\left(1 - e^{-at}\right) \end{array}}$

$$\dot{u}_a + \frac{1}{RC} u_a = \frac{1}{RC} u_e$$

Lösung: $u_a(t) = \frac{1}{\cancel{RC}} \cancel{RC} \left(1 - e^{-t/RC}\right).$

Bild (Python)

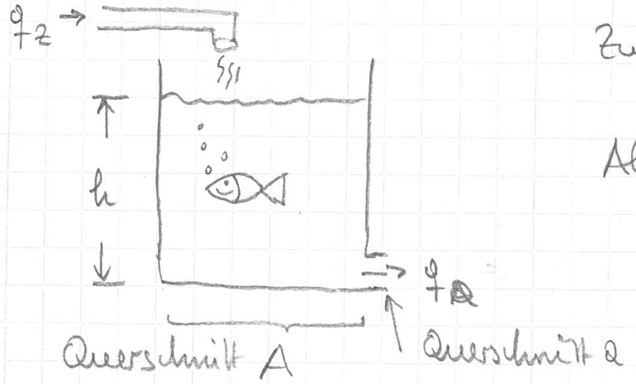


steady state: $t \to \infty$: $u_a(t) = 1$

Ist auch aus der DGL erkennbar durch setzen von $\dot{u}_a = 0$: $u_a = u_e = 1$.

Alternative Lösungsmethoden: Laplacetransformation, Exponentialansatz + part. Lsg., Trennung d. Variablen, exakte DGL, oder numerisch.

▷ Wasserstand



Zufluss q_z ... $\left[\frac{\text{Vol.}}{\text{Zeit}} \right]$

Abfluss q_a ... $\left[\frac{\text{Vol.}}{\text{Zeit}} \right]$

$$\underbrace{\frac{q_a}{a}}_{\text{Abflussgeschw.}} = \sqrt{2gh}$$

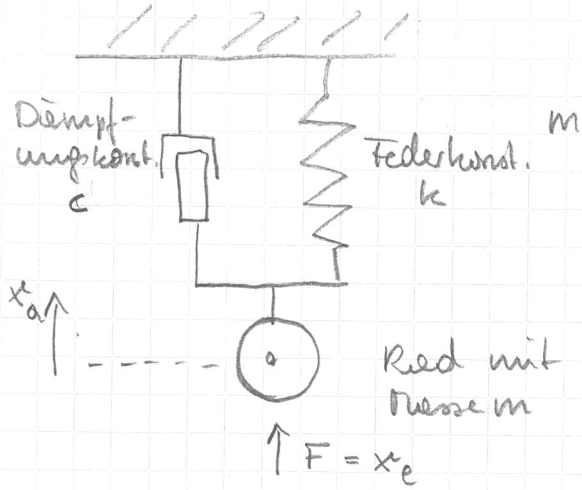
$$\frac{dV}{dt} = A \frac{dh}{dt} = q_z - q_a$$

$$\dot{h} = \frac{1}{A}(q_z - q_a) = \frac{1}{A} q_z - \frac{1}{A} a \sqrt{2gh}$$

$$\underline{\dot{h} + \frac{a}{A}\sqrt{2g}\sqrt{h} = \frac{1}{A} q_z}$$

... nicht-lineare, inhomog. Dgl 1. Ordg.

▷ Radaufhängung

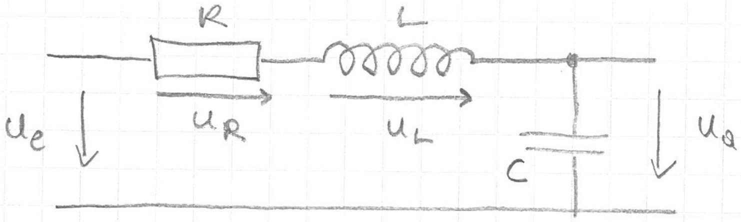


$$m \ddot{x}_a = -c \dot{x}_a - k x_a + x_e$$

$$\underline{m \ddot{x}_a + c \dot{x}_a + k x_a = x_e}$$

Schwingungsglg.: 2. Ordg., linear, inhom.

▷ RLC-Glied



$$u_e = u_R + u_L + u_a$$

$$u_R = iR$$

$$i = C\dot{u}_a$$

$$u_L = L\frac{di}{dt} = LC\ddot{u}_a$$

$$\Rightarrow \underline{u_e = RC\dot{u}_a + LC\ddot{u}_a + u_a}$$ ... lin., inhom. DGL 2. Ordg.

Schwingungsglg.

- Lösung: analytisch: - Exponentialansatz + partik. Lsg.
  - Laplacetransformation
  - Umschreiben zu einem System von 2 lin. GDGL → Eigenwerte u. -vektoren

  numerisch: ↳ odeint in Python

- steady state: $\dot{u}_a = 0,\ \ddot{u}_a = 0 \Rightarrow u_a = u_e =$ konst.
- Umschreiben in ein System für odeint:

$$\dot{u}_a =: v_a$$

$$\ddot{u}_a = \dot{v}_a = -\frac{1}{LC}u_a - \frac{R}{L}v_a + \frac{1}{LC}u_e$$

$$\begin{pmatrix} \dot{u}_a \\ \dot{v}_a \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 1 \\ -\frac{1}{LC} & -\frac{R}{L} \end{pmatrix}}_{A} \begin{pmatrix} u_a \\ v_a \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{LC} \end{pmatrix} u_e$$

$$\dot{x} = A\,x + b \cdot u_e$$

mit Zustandsvektor $x := \begin{pmatrix} u_a \\ v_a \end{pmatrix}$.